Executive Summary of the Minor Research Project Entitled

A STUDY ON

LATTICES RELATED TO
L-FUZZY TOPOLOGICAL SPACES

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Through this study we could identify the lattice structure of the $L-$topologies given by the level decompositions of a given $L-$fuzzy topological space, order relation being set inclusion. In this lattice we note that the join of any finite collection of elements is the smallest $L-$topology in the lattice containing their union and meet is exactly their intersection. It is proved that this lattice is complete, but not distributive. Also, for any two elements a, b in L we introduce the notions like b-upper set of a, $F-$upper set of a and using them we could characterize atoms and dual atoms in the lattice of our study. Also we investigate the situations under which this lattice becomes complemented. Further it is noted that the surjectivity of the $L-$fuzzy topology is a sufficient condition for the lattice under study to be isomorphic with L, but not necessary.

We have also investigated on the covers in the lattices of fuzzy topologies and $L-$topologies through simple extensions. If g is not a characteristic function, then we prove that no simple extension determined by g can be the discrete fuzzy topology. As a consequence of this we get certain well known results. However the above result is no longer valid in the $F-$lattice set up. So we introduce the concept of attainable

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$F-$lattice and extend the result with minimum modifications. Consequently if L is attainable, then on a set with at least three elements, the lattice of $L-$topologies has no dual atom. Certain properties of the lattices of $L-$topologies determined by the families of Scott continuous functions for a given topological space are investigated. The authors disprove certain known theorems on the above lattices and the correct results are furnished using the concept of strict chain join-generability in lattices.

This study also concentrates on the level decompositions corresponding to the elements in the range of an $L-$fuzzy topology on a given set and an investigation into the lattice structure of the same. In general, this lattice is not complete and distributive. However, certain necessary and sufficient conditions for it to be modular, distributive, complete and complemented are derived. Atoms and dual atoms of it along with the conditions for their existence are obtained. Certain related properties of it are also discussed.